Blue-noise Dithered Sampling

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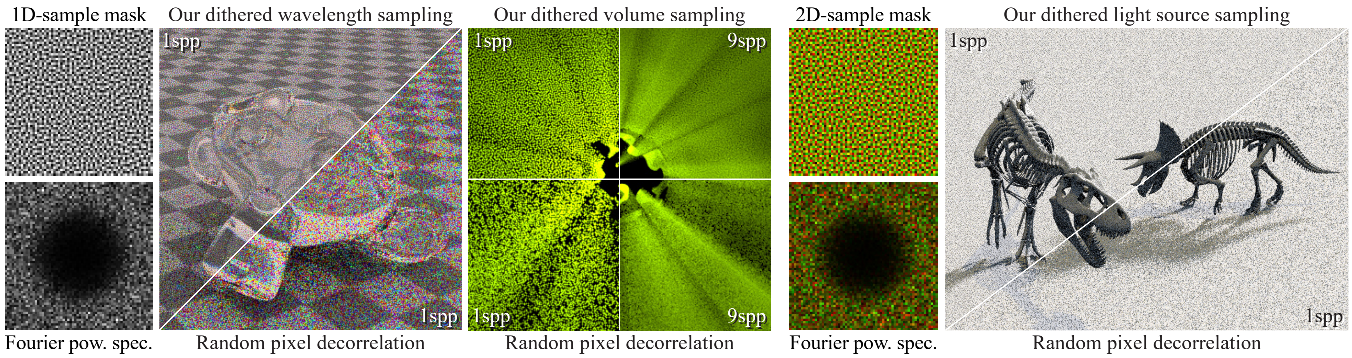

Figure 1: *Our method uses blue-noise dither masks tiled over the image to correlate the samples between pixels and thus minimize the low-frequency content in the distribution of the estimation error. Without actually reducing the amount of error, this correlation produces images with higher visual fidelity than traditional random pixel decorrelation, especially when using a small number of samples per pixel (spp).*

Keywords: Monte Carlo, sampling, blue noise, dithering

Concepts: •**Computing methodologies** → *Rendering;*

The visual fidelity of a Monte Carlo rendered image depends not only on the magnitude of the pixel estimation error but also on its distribution over the image. To this end, state-of-the-art methods use high-quality stratified sampling patterns, which are randomly scrambled or shifted to decorrelate the individual pixel estimates.

While the white-noise image error distribution produced by random pixel decorrelation is eye-pleasing, it is far from being perceptually optimal. We show that visual fidelity can be significantly improved by instead *correlating* the pixel estimates in a way that minimizes the low-frequency content in the output noise. Inspired by digital halftoning, our *blue-noise dithered sampling* can produce substantially more faithful images, especially at low sampling rates.

Blue-noise dithering. In digital halftoning, dithering is the intentional application of noise to randomize the error from quantizing a continuous-tone image [Lau and Arce 2008]. An efficient approach is to threshold the pixels using a blue-noise mask, which is a 2D array of scalar values arranged such that the Fourier power spectrum of any thresholded gray-level is isotropic and devoid of low frequencies. That is, neighboring pixels get very different thresholds, and similar thresholds are assigned to pixels far apart.

Dithered sampling. Our idea is to apply the concept of dithering to correlate pixel estimates in Monte Carlo distribution ray tracing. Given a d-dimensional sampling pattern, we toroidally shift it for every pixel, but rather than choosing the offset randomly, as done traditionally, we look it up in a *blue-noise sample mask* tiled over the image. The value of every pixel in such a pre-computed mask is a d-dimensional vector, and for $d = 1$ the mask is very similar to a halftoning mask. In this setting, traditional random-offset pixel decorrelation is equivalent to using a white-noise sample mask.

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When using a blue-noise sample mask, neighboring pixels evaluate different locations in the sampling domain, yielding a high-frequency noise distribution in the rendered image. Mitchell [1991] also observed that this is a desirable property and proposed a sample distribution optimality condition along with a simple algorithm to approximate it. Our approach aims to efficiently meet this condition and improve over Mitchell's algorithm.

Sample mask construction. Our blue-noise sample masks are scene-independent and can be built offline. Starting from a random (white-noise) mask M, we repeatedly swap random pixel pairs to minimize the following energy function via simulated annealing:

$$E(M) = \sum_{p \neq q} E(p,q) = \sum_{p \neq q} \exp\left(-\frac{\|p_\mathrm{i} - q_\mathrm{i}\|^2}{\sigma_\mathrm{i}^2} - \frac{\|p_\mathrm{s} - q_\mathrm{s}\|^{d/2}}{\sigma_\mathrm{s}^2} \right).$$

The energy function is a product of image- and sample-space Gaussians, where p and q are pixels, p_i and q_i are their integer 2D coordinates, and p_s and q_s are their associated d-dimensional sample values. Image-space distances are computed on wrapped boundaries so that the mask can be tiled seamlessly over the image. Following Ulichney [1993] we use $\sigma_\mathrm{i} = 2.1$, and for the sample-space Gaussian we set $\sigma_\mathrm{s} = 1$. The $d/2$ exponent corrects for the difference in the average distance between points in the d-dimensional sample space and the 2D image space.

Results. Figure 1 shows our 1D- and 2D-sample dither masks along with their corresponding Fourier power spectra. We also demonstrate how these masks compare against random pixel decorrelation for offsetting patterns with 1 and 9 samples. Thanks to the blue-noise error distribution, the images produced by our approach appear less noisy, even though the *amount* of estimation error is the same as that of traditional white-noise decorrelation. Note that we plot masks of size 64^2, but we used size 128^2 to render our images.

References

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